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The Influence of Enhanced Closed-loop Sensitivity Towards Breathing-type Structural Damage

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Abstract

This paper investigates the performance of a nonlinear damage detection method using sensitivity enhancing control (SEC). Damage nonlinearity due to the cyclic behavior of crack breathing could provide valuable evidence of structural damage without information of the structure's original healthy condition. Not having such information is considered a major challenge in vibration-based damage detection. In this study, two different categories of damage detection methods are investigated: frequency and time-domain techniques focusing on the benefit of SEC for breathing-type nonlinear damage in a structure. Numerical simulations using a cantilevered beam and spring-mass-damper system demonstrated that the level of nonlinear dynamic behavior heavily depends on the closed-loop pole placement through feedback control. According to SEC theory, the characteristic of the feedback gain defines the sensitivity of modal frequency to the change of stiffness or mass of the system. The sensitivity enhancement by properly designed closed-loop pole location more visually clarifies the evidence of crack nonlinearity than the open-loop case where no sensitivity is enhanced. A damage detection filter that uses time series data could directly benefit from implementing SEC. The amplitude of damage-evident error signal of the closed-loop case significantly increases more than that of the open-loop case if feedback control or SEC properly modifies the dynamics of the system.

Keywords: Damage detection; Breathing crack; SEC (Sensitivity Enhancing Control)

1. Introduction

Diagnosing structural integrity by observing the deviation of dynamic properties from the healthy condition of the structure has been widely known as vibration-based damage detection (Sohn et al., 2003). Two parallel approaches are used in vibration-based methods: frequency-domain and time-domain techniques. Which method is used depends on how the damage-sensitive feature is extracted from vibration responses. In frequency-domain algorithms, damage-indicative signatures such as the variation of modal frequencies, mode shapes, and/or frequency response functions are used for detecting and isolating damage in a structure (palacz and krawczuk, 2007). In time-domain algorithms, on the other hand, the residual or error signal, which is induced by the presence of damage or defects, is typically exploited by using time-history data (Liberatore et al., 2002). Both approaches have pros and cons in practice. Although the algorithm itself can be easily implemented, the frequency-domain method suffers from poor sensitivity towards small damages. Additionally, there is an inherent limitation of the linear model assumption. Time-domain methods are usually capable of capturing the onset of damage and accommodating the non-linearities of structural damages. A major drawback, however, is the computational challenges that arise as it keeps track of a considerable amount of

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time series data. Basically, vibration-based techniques are a type of model-based method, meaning a pair of conditions, i.e., pre and post-damaged states, must be compared. Thus, an accurate knowledge of the pre-damaged state that best represents the original healthy condition of a structure becomes an essential baseline for successful damage detection. Most vibration-based damage detection methods are used on the premise that an accurate pre-damaged model exists, but pre-damaged baseline information is not always readily available. Moreover, damage or crack in a structure is not the only reason that modal property changes. Quite often, the boundary conditions of actual structures can be harmlessly modified by environmental factors, such as stress relaxation, absorbed moisture, temperature variation, wear, or even regular maintenance. Sometimes, the state of the structural condition, healthy or damaged, is difficult to determine simply by detecting the change of modal parameters. Therefore, undisputed evidence of damage presence in a structure that does not necessarily rely on accurate knowledge of the structure's healthy state is highly desirable in the development of a robust damage detection method. For this reason, a non-model-based approach that does not require test data or analytical model of a structure's healthy state has gained attention in recent years. One of these approaches exploits the nonlinear behavior of a structure having a fatigue surface crack. A fatigue surface crack typically exhibits in a crack opening and closing, or simply called crack breathing (Douka and Hadjileontiadis, 2005).

Most of the research in damage detection has been focused on identifying open-type cracks. In general, the stiffness of crack region is assumed to remain intact during oscillatory motion of the structure. It is presumed that two surfaces of a crack do not contact each other even in a compressive bending mode. Prevailing examples of open-crack modeling include a simple reduction of thickness or degradation of elastic modulus in one of the structural members both in experiment and numerical simulation. Because of their simplicity in modeling the effect of reduced stiffness, most vibration-based damage detection methods exploit the change of modal frequency before and after the occurrence of open-crack damage. However, several reports have indicated that crack closing or crack breathing tends to reduce the amount of modal frequency shift. Cheng *et al.* (1995) demonstrated that the open-crack assumption might underestimate the true severity of a fatigue crack. The study discourages the use of the frequency shift of an open-crack as a basis for damage detection for it may result in a serious consequence. In this regard, Chondros *et al.* (2001) also pointed out that the open-crack model may provide a misleading conclusion about the amount of frequency drop in an aluminum beam when a breathing fatigue crack exists in the beam. Kisa and Brandon (2000) propose a successive modal transformation method to avoid the transition process from the frequency-domain to the time-domain in modeling the contact condition between crack surfaces. Their study also verified that the value of modal frequency of a beam having a breathing crack generally lies between the modal frequency of the healthy beam and that of a beam having an open-crack. Chati, *et al.* (1997) investigated the idea of effective frequency, which represents the associated bilinear mode of a piecewise-linear system. In the study, eigenfrequencies of the bilinear 2-DOF system are defined and verified by a perturbation method. Pugno *et al.* (2000) considered a problem of detection for a beam with multiple breathing cracks. Tsyfansky and Beresnevich (2000) showed that the evidence of nonlinear vibration such as a super-harmonic resonance can be a potentially more sensitive indication than modal frequency shifts in the detection of fatigue cracks in aircraft wings. Although numerous damage detection studies still use open-crack models, the notion that a crack always remains open is an unrealistic assumption. In practice, crack opening/closing or breathing is a more general criterion of a structural damage. Furthermore, nonlinearity of damage is not only limited to the fatigue crack case. Under cyclic loading, mechanical devices such as loosened bolts, couplings, press-fitted and riveted joints exhibit nonlinear dynamic behavior similar to a beam with a fatigue-breathing crack (Leonard et al., 2001).

Another undesirable aspect of a fatigue crack-basis of damage detection is the extremely low sensitivity toward modal parameters. Because fatigue cracks usually propagate with no substantial deformation, even a small depth of a crack could result in a fatal consequence. As a result, it is crucial that the presence and the location of small cracks in structures be promptly identified. However, the general level of sensitivity of modal frequency towards small damage could be an issue in the practical application of damage detection in the presence of environmental uncertainties and measurement errors. There has been research on the detection and localization of structural damage by enhancement of sensitivity of modal frequency through feedback control, so-called Sensitivity Enhancing Control (SEC) (Ray and Tian, 1999; Ray et al., 2000; Koh and Ray, 2004). The motivation of SEC in frequency-domain damage detection lies in the case where signal processing fails from low sensitivity to discern the frequency shift of a damaged structure due to measurement noise or environmental disturbance. This low sensitivity issue can be overcome only by changing the structure's dynamics to enhance the sensitivity of modal parameters towards damage. Ray and Tian first proposed this approach to improve the sensitivity of closed-loop natural frequency to stiffness and mass damage (Ray and Tian, 1999). In the study, it is shown that sensitivity to thickness reduction at the root of the cantilevered beam increases by a factor of approximately forty (first mode) and a factor of five (third mode). Ray *et al.* (2003) also investigated the practical application of SEC in detecting a fatigue crack in a plate. In the study, SEC significantly improves damage detection by increasing the sensitivity of modal frequencies to a realistic fatigue surface crack. While the conventional open-loop approach struggles to distinguish damage from measurement noise due to low sensitivity, SEC successfully detects damage in a smart structure with noticeably increased sensitivity margins. The previous study showed that SEC in the SDOF system can be easily extended to a cantilevered beam structure (Ray et al., 2000; Koh and Ray, 2004). The SEC moved the first four modes of the FE beam model having 8 elements to the frequencies slightly lower than those of the open-loop case. SEC increases the frequency shifts under 10% reductions of the Young's modulus in the first element (closest to the root) of the beam. The study showed that as the closed-loop frequencies of each mode decrease, the frequency shifts increase, illustrating sensitivity enhancement (Koh and Ray, 2004). Because the main framework for SEC is based on the sensitivity of the closed-loop pole in linear systems, the feasibility and performance of SEC in nonlinear damage detection has never been discussed.

The major difference between time-domain and frequency-domain techniques is the nature of damage-evident dynamic quantities. Time-domain methods directly utilize digitized time series data from sensors, without going through signal processing or more specifically, a numerical Fourier transform. Thus, the original nature of raw data can be preserved, and the linearity assumption is not required. Although there are several diagnostic techniques that exploit the time series data directly measured from a damaged structure, fault detection filters or analytical redundancy methods are widely known for real-time damage monitoring. Kranock (2000) investigated an observer-style damage detection filter while Seibold *et al.* (1996) proposed a method using a bank of Kalman filters to identify the depth of a fatigue crack in a rotor system. Recently, Koh *et al.* (2005a) introduced a mathematical framework for an input error function which generates nonzero error signals as the structure experiences structural de-gradation in real-time. The concept of the input error function, derived from the modified interaction matrix formulation, was originally developed for solving actuator failure detection problem (Koh et al., 2005b). This time-based technique is capable of capturing the initiation and locations of multiple damages simultaneously.

Given a breathing-type crack, this paper investigates the performance of two different damage detection approaches: one using frequency-domain data and the other using time-domain data. Especially, a comparative study is done to evaluate the influence of SEC on both approaches. Numerical simulations show that damage nonlinearity due to crack breathing is significantly related to the placement of closed-loop poles in the complex plane. The nature of damage is a breathing-type crack that alternately switches between two stages, i.e., crack opening and closing. The dynamic behavior of this damage is inherently nonlinear due to the switching process and, thus, an abrupt change in local stiffness is induced by the opening and closing of fatigue crack surfaces. Thus, the author evaluates the effect of SEC-implemented closed-loop system on the detectability of nonlinear damage in a structure. Three different numerical models are considered for discussions in this paper. The first one is a single-DOF cantilevered beam where the stiffness reduction caused by a surface crack is simulated through a theory in fracture mechanics. The second model is an eight-element or equivalently 16-DOF cantilevered beam that has the same dimensions and material properties of first model, i.e., single-DOF cantilevered beam. Lastly, a three-DOF spring-mass-damper system is considered for damage detection filter. Numerical simulations have shown that SEC is effective in solving damage detection problems in both frequency-based and time-based techniques.

2. Description of breathing crack

Figure 1 illustrates two different geometrical states of a crack under general dynamic loading, which is more obviously distinguished by the incident of contact between two crack surfaces. In this section, the nonlinear dynamic behavior of a breathing crack previously investigated by Cheng *et al.* (1995) is partially reviewed for illustration. The study proposed a super or sub-harmonic peak in the frequency response function as an indication of the breathing crack, which is shown in the simulation result of a simplified single DOF beam model. For verification, a cracked beam having the same dimension and material properties is considered, as shown in Fig. 2 and Table 1. The value of open-crack stiffness (k_o) is derived from the flexibility of the cracked beam, which is based on the Paris equation in fracture mechanics (Cheng et al., 1995). On the other hand, the stiffness of the closed-crack (k_h) beam is considered to be the same as that of the intact (healthy) beam. Under the assumption that the cycle of crack opening and closing coincides with the frequency of the external sinusoidal loading, the stiffness value of the cracked beam varies with time. This sinusoidal variation of stiffness value occurs because the crack opening and closing is not precisely a piecewise transition but rather is close to continuous transition due to the rough interference between the crack surfaces. The diagram in Fig. 3 illustrates the transition of the stiffness value from open-crack to closed-crack. Here, the radius of circle, *R,* is defined as the amplitude of the stiffness change, Δk . Hence, the dynamic equation of motion is expressed as,

$$m\ddot{x} + c\dot{x} + \left[\frac{k_h + k_o}{2} + \frac{k_h - k_o}{2}\cos\omega t\right]x = F\sin\omega t \quad (1)$$

where *m*, *c*, and *F* indicate mass, damping, and excitation force, respectively. Note that generalized mass and stiffness parameters are used to represent the cantilevered beam as a single DOF system (Cheng et al., 1995).

$$k_h = \frac{EI\pi^4}{32L^3}, \qquad m = 0.228m'L \quad (2)$$

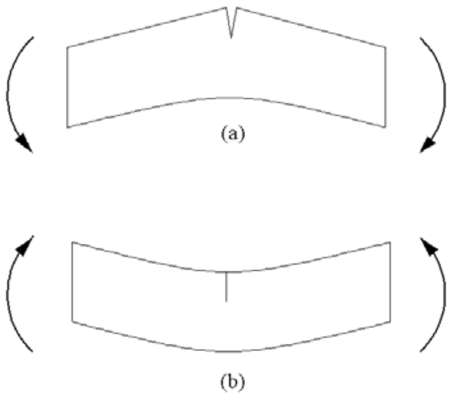

Fig. 1. Schematics of crack *opening* (a) and crack *closing* (b).

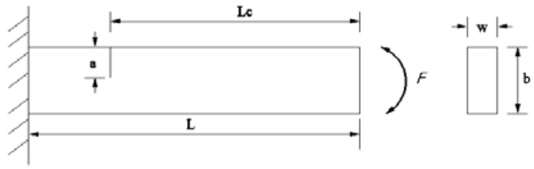

Fig. 2. The geometry of a cantilevered beam having a fatigue surface crack.

Table 1. Dimensions and material properties of the cantilevered beam.

L	Lc	b	w	a	E (Young's modulus)
9 *m*	8.1 *m*	0.26 *m*	0.15 *m*	0.2×b	206×10^9 N/m^2

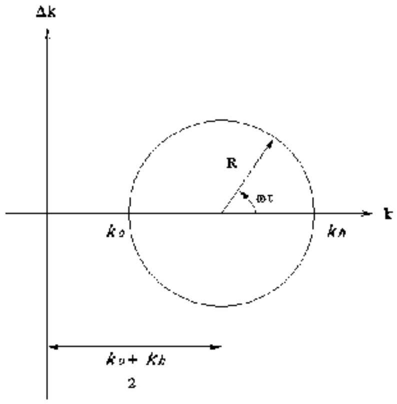

Fig. 3. The visual representation: the stiffness value as a function of forcing function.

Given arbitrary initial conditions, Eq. (1) is solved numerically by using the Runge-Kutta method. Finally, the frequency response function is obtained from Fast Fourier Transform. Figure 4 shows the comparison between open-crack and breathing crack in both time and frequency responses given that the

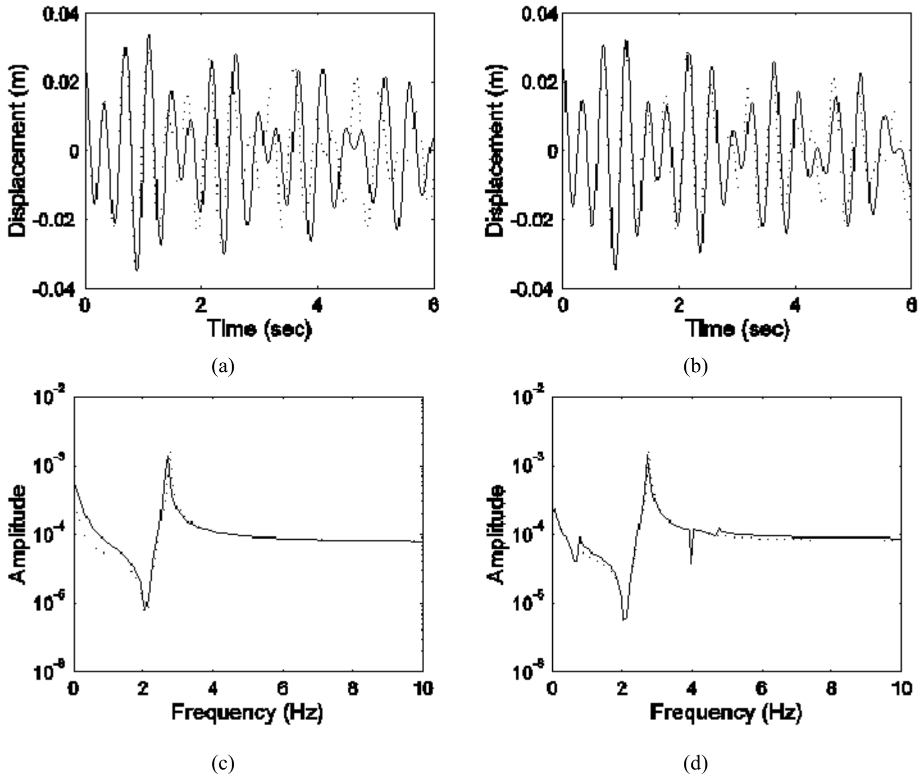

Fig. 4. The comparison of time and frequency responses between open and breathing crack systems given crack breathing frequency of 2 Hz: (a) time response of open crack, (b) time response of breathing crack, (c) frequency response of open crack, (d) frequency response function of breathing crack for damaged forced vibration: undamaged (---), damaged (—).

crack breathing frequency is generated by using a 2 Hz sinusoidal input ($F\sin\omega t$). Here, Fig. 4(a) and (c) indicate open-crack responses, while (b) and (d) exhibit responses of the cantilevered beam having abreathing crack. Clearly, the nonlinearity induced by crack opening and closing generates super/sub harmonics or side peaks around the resonance frequency of the beam, as shown in Fig. 4(d). This side peak can be a unique indication of damage in a structure; in other words, the natural frequency does not have to be compared with that of a healthy structure.

3. Sensitivity Enhancing Control (SEC)

The basic idea of Sensitivity Enhancing Control (SEC) is to increase the modal sensitivity to stiffness and mass damage by using feedback control within the framework of a smart structure (Ray and Tian, 1999). SEC drives the poles of the closed-loop smart structure to the location in the complex plane where modal frequency shifts are more sensitive to damage. The concept of sensitivity enhancing control can be easily demon-strated in a spring-mass or a Single-Degree-of-Freedom (SDOF) system. The sensitivity of natural frequency (ω_n) in SDOF to small changes in stiffness (k) and mass (m) is defined as Eq. (3).

$$\frac{\partial\omega_n}{\partial k}=\frac{\omega_n}{2k},\qquad \frac{\partial\omega_n}{\partial m}=-\frac{\omega_n}{2m} \tag{3}$$

Hence, the smaller the stiffness and mass, the larger the sensitivity of natural frequency to changes of a structure. If the system is controlled by state feedback, the proper selection of a control gain matrix $G=\left[G_1\ \ G_2\right]$ arbitrary modifies the closed-loop natural frequency.

$$\omega_{n,cl}=\sqrt{\frac{k+G_1}{m}} \tag{4}$$

Accordingly, the sensitivity of closed-loop natural frequency ($\omega_{n,cl}$) can be enhanced by the value of the control gain G_1 :

$$\frac{\partial \omega_{n,cl}}{\partial k} = \frac{\omega_{n,cl}}{2(k+G_1)} \tag{5}$$

For higher order and multi-variable systems, feedback control design requires a more systematic inverse eigenvalue technique such as pole placement. The pole placement problem for the system can be formulated as follows. Given a set of n complex poles P, and a matrix G, such that the eigenvalues of $A+BG$ are P, the matrix G is denoted to as the feedback gain, since if $u = Gx$, then Eq. (4) becomes the natural frequency of the closed-loop system. In a Single-Input Pole Placement (SIPP) problem, the feedback gain vector G always exists and is unique for all sets of P of distinct poles if and only if (A, B) is controllable. However, SIPP is quite sensitive to the perturbation of system parameters. The traditional goal of pole placement requires that the implemented poles of the closed loop system are close to the desired ones. If the desired poles of the exact closed-loop system are sensitive to perturbations, then this goal cannot be guaranteed (Xu and Mehrmann, 1988). SEC exploits this weakness of pole placement in a positive way, thereby magnifying the variation of system parameters. In the multi-input case, pole placement is essentially an underdetermined problem with many degrees-of-freedom so that the sensitivity to system perturbations is not uniquely determined by pole locations. Specific criteria (robustness or sensitization) may be used to restrict these degrees-of-freedom in the pole assignment problem. Although sensitivity enhancement in the multi-input case may require an additional optimization process in the design of control laws, multi-input SEC can be a more effective tool for maximizing sensitivity.

4. Implementing SEC for breathing crack simulation

This section deals with the numerical simulation of a multi-DOF cantilevered beam having a breathing crack. Here, SEC is implemented to investigate the effect of closed-loop pole sensitivity toward the nonlinearity of breathing-type damage. In the multi-DOF system case, the cycle of crack opening and closing does not necessarily coincide with the excitation frequency. Hence, to simulate the behavior of crack-closure in higher modes, the governing equation of motion should be directly integrated for every time step. In other words, the stiffness value of the healthy part of a structure should be readily switched to the value presumed for the damaged structure at every integration step. This switching sequence is determined by the rotational angles in neighboring DOFs of the damaged element.

An eight-element cantilevered beam model is used for the breathing crack simulation. The dimension and material properties are the same those shown in Fig. 2 and Table 1. The damage severity and location are equivalent those of a 1% thickness reduction of the first element from the clamp. The simulation is performed by direct time integration at time step of 0.2ms using the central difference scheme. The time step is predetermined by the smallest period of the finite element model necessary to meet the condition for stability of numerical integration. In order to implement the effect of a breathing crack, the switching sequence of the stiffness matrix is made to occur as a function of the sign change at the rotational DOFs around the damaged element. More specifically, during the time integration, if the rotational DOF (curvature) of the first element becomes negative, indicating that the crack is open, the local stiffness matrix of that element is switched to the damaged stiffness matrix.

For a demonstration of SEC, the first three modes are controlled by full-state feedback, and two different control laws are implemented. The first control law is designed to enhance the closed-loop pole sensitivity for stiffness damage. The second one is designed to diminish the sensitivity. Thus, two control laws are considered in this simulation: one for enhancing and the other one for reducing sensitivity (SRC: Sensitivity Reducing Control) toward stiffness damage. The natural frequencies of the open-loop and two closed-loop systems are shown in Table 2. The feedback gain matrix multiplies the velocity and displacement of each DOF and then the resultant vertical force is applied to the DOF at the first element

Table 2. Natural frequencies (Hz) of the open-loop and closed-loop (SEC, SRC) systems of the cantilevered beam, SEC: sensitivity enhancing control, SRC: sensitivity reducing control.

Mode	Cantilevered beam		
	Open-loop	Closed-loop (SEC)	Closed-loop (SRC)
1	0.52	0.35	0.67
2	3.22	2.42	4.01
3	9.01	7.41	10.6

from the root of the beam. Finally, the frequency response is sought by transforming the impulse time responses to frequency spectra by using Power Spectral Density (PSD).

4.1 Simulation results

Figure 5 shows the simulation result of PSD for a cantilevered beam having a breathing crack. The beam is excited by an impact force of 100 N at the tip and displacement at the same location is recorded with a sampling time of 0.2 ms. Specifically, Fig. 5(a) shows the PSD of an open-loop system, while Fig. 5(b) illustrates the simulation result of the SEC-implemented, closed-loop system, whose first three closed-loop poles are lowered in a complex plane. On the other hand, Fig. 5(c) presents the PSD of a closed-loop system, whose poles are elevated in the complex plane. Hence, the purpose of designing each control system is different: a closed-loop system (b) is designed for enhancing the sensitivity the stiffness damage, while the closed-loop system (c) is for reducing the sensitivity of stiffness damage.

Here, open-loop systems are compared to two different closed-loop systems to investigate the influence of closed-loop pole locations on the crack nonlinearity in the frequency-domain. The simulation showed that multiple side peaks occur between resonant frequencies, a result which is similar to the result of the previous single-DOF system, as shown in Fig. 4(d). More visually distinguishable side peaks are observed in the closed-loop system [Fig. 5(b)], which is designed to increase the modal sensitivity. Also, for

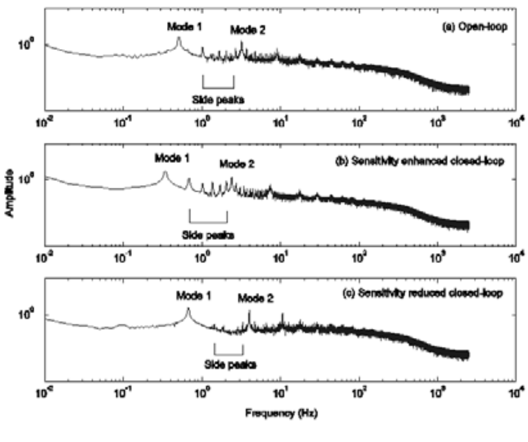

Fig. 5. Comparison of power spectral density between (a) open-loop, (b) sensitivity-enhanced, and (c) sensitivity-reduced closed-loop systems having a breathing crack.

the case of the sensitivity-reduced closed-loop system [Fig. 5(c)], the magnitudes of side peaks were significantly diminished, even compared to the open-loop response, as shown in Fig. 5(a). Similar to the modal frequency sensitivity, crack nonlinearity is also affected by the location of the closed-loop poles. SEC, which improves the result of non-model-based damage detection, can magnify side peaks induced from crack nonlinearity.

4.2 Real-time damage detection filter

This section investigates the influence of SEC on the time-domain damage detection method, especially when the structure experiences breathing-type nonlinear damage similar to that of the frequency-domain case described in the previous section. Recently, we introduced a mathematical framework for a damage detection filter, which exploits the concept of input error function in monitoring incipient damages in a structure (Koh et al., 2005a). Here, the performance of the damage detection filter for a breathing-type crack will be compared between an open-loop and an SEC-implemented closed-loop system. Although the details of the filter design can be found in reference (Koh et al., 2005a), the underlying theory will be briefly explained here for completeness. Since the input error function is developed in state-space framework, the second-order equation of motion should be expressed in first-order form. Consider an n-th order, r-input, m-output discrete-time model of a system in the state-space format

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{6}$$

By repeated substitution for some $p \geq 0$,

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{C} u_p(k) \\ y_p(k) &= Ox(k) + Tu_p(k) \end{aligned} \tag{7}$$

where $y_p(k)$ and $u_p(k)$ are defined as column vectors of input and output data going p steps into the future starting with $y(k)$ and $u(k)$, respectively

$$\begin{aligned} y_p(k) &= \left[y(k) \;\; y(k+1) \;\; \cdots \;\; y(k+p-1) \right]^T, \\ u_p(k) &= \left[u(k) \;\; u(k+1) \;\; \cdots \;\; u(k+p-1) \right]^T \end{aligned} \tag{8}$$

For sufficiently large p, $\mathcal{C}$ is an extended $n \times pr$ controllability matrix, O is an extended $pm \times n$ observability matrix, and T is a $pm \times pr$ "Toeplitz" matrix of the system of Markov parameters.

$$\mathcal{C} = \left[A^{p-1}B, \ \dots \ ,AB,B\right] \ , \ O = \left[C,CA, \ \dots \ ,CA^{p-1}\right]^T ,$$

$$T = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & \cdots & \cdots & 0 \\ CAB & CB & D & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 \\ CA^{p-2}B & \cdots & CAB & CB & D \end{bmatrix} \quad (9)$$

Rewriting the Eq. (7) in terms of the contribution of each individual input $i = 1, 2, \cdots, r$

$$\begin{aligned} x(k+p) &= A^p x(k) + \sum_{i=1}^{r} \mathcal{C}_i u_i^{(p)}(k) + \sum_{i=1}^{r} \mathrm{B}_i u_i(k+p-1) \\ y_p(k) &= Ox(k) + \sum_{i=1}^{r} T_i u_i^{(p)}(k) + \sum_{i=1}^{r} \mathrm{D}_i u_i(k+p-1) \end{aligned} \quad (10)$$

where $B = \left[B_1, B_2, \cdots, B_r\right]$ and $\mathcal{C}_i = \left[A^{p-1}B_i, \cdots, A^2B_i, AB_i\right]$.
Also

$$u_i^{(p)}(k) = \begin{bmatrix} u_i(k) \\ u_i(k+1) \\ \vdots \\ u_i(k+p-2) \end{bmatrix} \text{ and}$$

$$\mathcal{C}_i = \begin{bmatrix} D_i & 0 & \cdots & 0 \\ CB_i & D_i & \ddots & \vdots \\ CAB_i & CB_i & D_i & \vdots \\ \vdots & \ddots & \ddots & D_i \\ CA^{p-2}B_i & \cdots & CAB_i & CB_i \end{bmatrix} \quad (11)$$

The concept of interaction matrix M_i is introduced to add and subtract the product $M_i y_p(k)$ to Eq. (10). In order to monitor the integrity of the i-th actuator, the relationship between the input and output should be established for each actuator. Specifically, certain input-output functions should generate error signals if and only if the i-th actuator has failed. Finally, the input error function for the i-th actuator can be derived as:

$$\begin{aligned} e_i(k) &= \alpha_0 y(k) + \alpha_1 y(k-1) + \cdots + \alpha_p y(k-p) \\ &\quad + \beta_1 \bar{u}_i(k-1) + \beta_2 \bar{u}_i(k-2) + \cdots \\ &\quad + \beta_p \bar{u}_i(k-p) \end{aligned} \quad (12)$$

Here, $\bar{u}_i(k)$ is the commanded input, such that $u_i(k) = \bar{u}_i(k) + z_i(k)$ where $z_i(k)$ is the input error or internal force caused by the variation of the system. Thus, one should monitor the value of $z_i(k)$ to see whether the system experiences unknown structural degradation. For an r-input and m-output system ($m \geq r$), each coefficient $\alpha_0, \alpha_1, ..., \alpha_p$ is a $1 \times m$ row vector, whereas each coefficient $\beta_1, \beta_2, ..., \beta_p$ is a scalar. Given a state-space model of the healthy structure, the coefficients of Eq. (12) can be easily obtained (Koh et al. 2005b). For structural damage monitoring, the non-zero error signal from Eq. (12) for the i-th actuator indicates the initiation of structural damage at the i-th stiffness member. Note that the number of outputs should be equal or greater than that of the actuator to have a valid input error function, as shown in Eq. (12).

4.3 SEC for damage detection filter

Here, the input error function will be used as the damage detection filter to detect breathing-type damage. Also, the results will be discussed to determine the effect of SEC on the performance of a time-domain damage filter. First, a 3 DOF spring-mass-damper model is considered, as shown in Fig. 6. The mass, damping, and spring constant of the system are shown in Fig. 6 in suitable units. Again, imposing two different spring constant values according to the relative position of the adjacent masses simulates a breathing crack. The time-varying behavior of a breathing crack is simulated by alternately changing the spring constant, as it experiences compression and expansion. For example, if the spring element k_3 undergoes breathing-type damage, the sign of the relative displacement between m_2 and m_3 determines

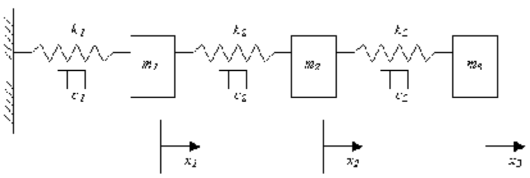

Fig. 6. Schematic drawing of 3 DOF spring-mass-damper system; m_1=2, m_2=3, m_3=3, k_1=30, k_2=25, k_3=35, c_1=c_2=c_3=0.1, in all suitable units.

the compression (closing) and expansion (opening) of the spring k_3.

Unlike the frequency-based methods, the time-domain damage detection filter can isolate the locations and the instant of damage in real-time. An impulse force (100 N) applies on m_3 to excite the spring-mass-damper system. Although the damage detection filter can accommodate any type of input, an impulse signal is used because it is easy to compare the effectiveness of SEC. It is assumed that a breathing-type damage occurred in spring k_3 when its value reduced by 20% after 50 seconds. Position measurements of all three masses were fed into the damage filter producing a nonzero error signal for the damaged spring element. Figures 7 and 8 show the simulation results of the position of each mass, as the spring k_3 experienced breathing-type damage. Figures 7 and 8 indicate open-loop and SEC-implemented closed-loop responses, respectively. Note that a gap between healthy and damaged response develops after 50 seconds. However, the location of damage is unidentifiable directly from time-domain responses. Interestingly, the closed-loop response exhibits a larger damage-induced deviation than the open-loop response does, indicating enhanced sensitivity between the healthy and the damaged state. Figures 9 and 10 show the profiles of the error function (Eq. 12) for both open-loop and closed-loop systems as they experience breathing-type damage for time span between 50 to 120 seconds. Apparently, the damage filter accurately identified the onset of damage and its location as they occurred after 50 seconds of excitation. Interestingly, the pattern of error signal explicitly indicated the bilinear behavior, which is uniquely induced by breathing-type damage in a system. Thus, the nonlinearity of damage can be easily identified from the pattern of the filtered error signal. It can be considered that the biased error signal is indicative of the emergence of side peaks in PSD, as shown in Fig. 5.

Similar to the frequency-based method, SEC is implemented through closed-loop pole placement of all three modes, but only the first mode is significantly modified to increase the sensitivity of damage detection. Table 3 shows the open-loop and SEC-implemented closed-loop pole locations in the complex domain. Damage sensitivity is increased differently from that of the frequency-based case discussed in the previous section. Unlike the frequency-based case, the natural frequency of the first mode increased. Figures 9 and 10 show that a closed-loop system increases the amplitude of the error signals more than an open-loop case does, indicating better detectability for capturing the onset of damage occurrence (Figures 9 and 10 are depicted in the same scale). As shown in Fig. 10, the maximum amplitude of the damage-induced nonzero error signal from the closed-loop system increased up to 3.5 times more than that of the open-loop system. Feedback control increased the deviation between the healthy and damaged states, resulting in higher amplitude of the error signal. In summary, unlike the open-loop system, the closed-loop system has a controller that can be specifically designed to maxi-mize the deviation between healthy and damaged states. However, the

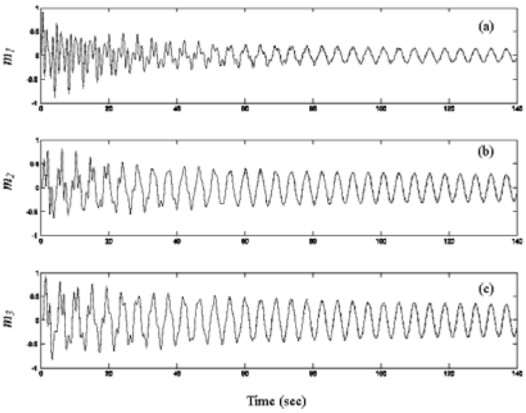

Fig. 7. Impulse response of open-loop spring-mass-damper system; (a) position of m_1, (b) m_2, and (c) m_3. (—): undamaged, (---): breathing-type damage on spring k_3.

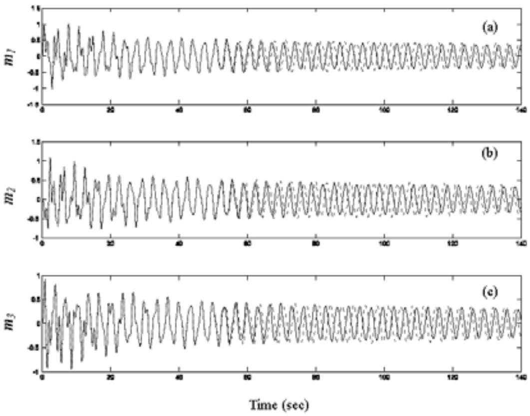

Fig. 8. Impulse response of SEC-implemented closed-loop spring-mass-damper system; (a) position of m_1, (b) m_2, and (c) m_3. (—): undamaged, (---): breathing-type damage on spring k_3.

Table 3. Pole locations of the open-loop and closed-loop spring-mass-damper systems.

Mode	Spring-mass-damper system	
	Open-loop	Closed-loop
1	-0.0035±1.39i	-0.0035±1.90i
2	-0.0315±4.53i	-0.0315±4.37i
3	-0.0651±6.06i	-0.0651±6.12i

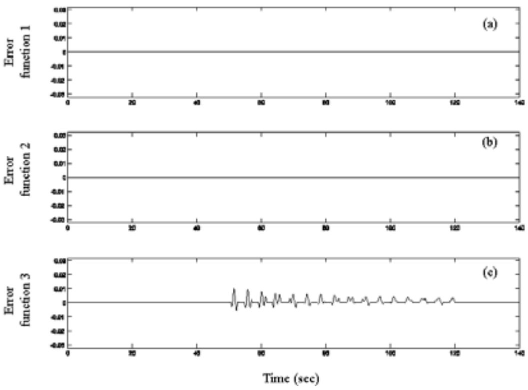

Fig. 9. Profiles of error function from damage detection filter on the open-loop spring-mass-damper system; (a) error function for spring k_1, (b) k_2, and (c) k_3. Breathing-type damage occurs on spring k_3 from 50 to 120 seconds.

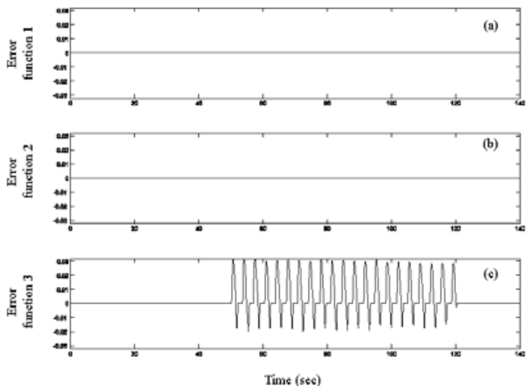

Fig. 10. Profiles of error function from damage detection filter on the SEC-implemented closed-loop spring-mass-damper system; (a) error function for spring k_1, (b) k_2, and (c) k_3. Breathing-type damage occurs on spring k_3 from 50 to 120 seconds.

enhancement of the amplitude of the filtered error signal by feedback control could be greatly affected by the location of the closed-loop poles in the complex plane. Simply reducing the natural frequency quency of the first mode does not guarantee a sensitivity-enhanced, filtered error signal in the time-domain. The explicit correlation between the location of closed-loop poles and the sensitivity of filtered error signal is still unknown due to the highly nonlinear aspect of an inverse eigenvalue problem, which is severely coupled with the multidimensional nature of damage locations and intensities. It should be also noted that in the context of explaining the nature of a damage detection filter, the benefit of the non-model based method is only limited to the frequency-domain method described in earlier sections.

5. Conclusions

This study integrates crack nonlinearity and closed-loop pole sensitivity, which allows non-model-based damage detection. Appropriate placement of closed-loop poles of a given system not only increases the sensitivity of modal frequency toward the linear perturbations of system parameters but also makes the evidence of crack nonlinearity more prominent than that of an open-loop case. Although the underlying theory of SEC originally focused on sensitivity enhancement of the modal frequency shift, the same framework can be applied to amplify the effect of breathing-type nonlinear damages in a structure. Enhanced closed-loop sensitivity also improves the detectability of breathing-type damage in the temporal-domain algorithm, which primarily uses time series data directly measured from sensors. For example, a damage detection filter using analytical redundancy of the system generates more powerful error signals when a properly designed closed-loop system experiences a breathing-type nonlinear damage. It is considered that the enhanced closed-loop sensitivity intensifies the nonlinear behavior of a system. This study is important because the non-linearity caused by crack breathing can be a valuable dynamic feature for diagnosing damages in a structure, especially when the baseline or healthy state information is unavailable. Numerical simulations of a cantilevered beam and a spring-mass-damper model show that a properly tuned SEC can successfully highlight the evidence of the presence of breathing-type damage in a structure to assist in the development of a more practical damage detection technique.

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